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Piloting Strategies for Controlling a Transport Aircraft After Vertical-Tail Loss

Götz Bramesfeld,* Mark D. Maughmer,† and Steven M. Willits‡
Pennsylvania State University, University Park, Pennsylvania 16802

In a number of instances, aircraft that have suffered in-flight damage to their airframe or control system were brought safely to the ground using unconventional means of control. The success in these cases depended greatly on the pilot having had some exposure to unconventional control strategies. Control strategies are considered for cases of aircraft with damage only to the primary control system, as well as cases in which the vertical tail is lost. The piloting strategies are developed using optimal control theory, which optimizes the control law for a desired maneuver and a chosen aircraft configuration. The results show that, despite the loss of the primary control system or of the vertical tail, control of the aircraft is often possible, although it requires the use of unconventional control strategies, in particular, of differential thrust. Especially in the case without a vertical tail, the maneuver in which adverse yaw induces a rolling moment opposite to the intended yaw direction is somewhat surprising and, initially, counterintuitive.

Nomenclature

$\boldsymbol{A}$ = plant matrix
$\boldsymbol{B}$ = control matrix
b = wing span
C_D = drag coefficient, drag/$\frac{1}{2}\rho V_\infty^2 S$
C_{D_α} = drag coefficient variation (with angle of attack), $\partial C_D/\partial\alpha$
C_L = lift coefficient, lift/$\frac{1}{2}\rho V_\infty^2 S$
C_{L_α} = lift coefficient variation (with angle of attack), $\partial C_L/\partial\alpha$
C_l = rolling moment coefficient, roll moment/$\frac{1}{2}\rho V_\infty^2 Sb$
C_m = pitching moment coefficient, pitch moment/$\frac{1}{2}\rho V_\infty^2 S\bar{c}$
C_n = yawing moment coefficient, yaw moment/$\frac{1}{2}\rho V_\infty^2 Sb$
C_Y = side force coefficient, side force/$\frac{1}{2}\rho V_\infty^2 S$
$\bar{c}$ = mean aerodynamic chord
$\boldsymbol{H}$ = weighting matrix (terminal)
J = performance index
$\boldsymbol{K}$ = Riccati matrix
l = roll moment
M = Mach number
m = pitch moment
n = yaw moment
P = roll rate; positive when right wing moves down
Q = pitch rate; positive when aircraft nose moves up
$\boldsymbol{Q}$ = weighting matrix (tracking)
R = yaw rate; positive when aircraft nose moves to right
$\boldsymbol{R}$ = weighting matrix (control effort)
$\boldsymbol{r}$ = desired state trajectory
S = wing area
$\boldsymbol{s}$ = command signal
T = thrust
U = forward velocity
$\boldsymbol{u}$ = control vector
V = side velocity; positive when aircraft moves to right
V_∞ = flight speed
W = downward velocity
$\boldsymbol{x}$ = state vector
α = angle of attack; positive when aircraft nose up
β = sideslip angle; positive when aircraft moves to right
δ_A = aileron angle; positive when right aileron up
δ_E = elevator angle; positive when trailing edge up
δ_R = rudder angle; positive when trailing edge to left (negative yaw, but positive roll moment)
Θ = pitch angle; positive when aircraft nose up
Φ = bank angle; positive when left wing up
Ψ = yaw angle; positive when aircraft nose points to right of flight path

Introduction

SINCE the advent of commercial aviation, incidents have occurred in which damage was done to the airframe and/or control system while the aircraft was in flight. In some of these cases, the crew was able to control the aircraft for a considerable time and even attempt a landing by applying unconventional control strategies. For example, considerable attention has been given to United Airlines Flight 232 from Denver to Chicago on 19 July 1986.[1] During cruise at 37,000 ft, a turbine disk failure in the tail-mounted engine of the DC-10 damaged several hydraulic lines in the vertical stabilizer. This caused a near complete loss of the conventional controls. With almost no aileron and little elevator control, the crew attempted an emergency landing in Sioux City, Iowa, by using differential thrust on the remaining two engines as the main controller. A wings level approach was maintained until, at about 50 ft above the runway, the left wing dipped down and struck the ground. In the crash, 185 of the 296 passengers survived. More recently, a DHL Airbus A300 cargo flight experienced a complete loss of hydraulics after it was struck by a missile after departing from Baghdad.[2] The pilots were able to stabilize the aircraft with the help of differential thrust and perform a successful emergency landing without any injuries or loss of life. An example of substantial structural damage is that of a B-52 that lost most of its vertical tail due to severe gusts after flying in the lee of two mountain peaks.[3] Despite the damage, the crew kept control of the aircraft and continued its mission for an additional 6 h. Less fortunate were the occupants of American Airlines Flight 587 (Ref. 4). After departing from LaGuardia, the aircraft encountered turbulence that possibly originated from a preceding airliner. Apparently, a combination of loads induced by gusts and control inputs resulted in the structural failure of the attachment of the vertical tail, which subsequently departed the aircraft. The pilots were not able to regain control, and the aircraft crashed into a populated area. This resulted in the loss of 265 lives, including 5 persons on the ground.

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*Instructor, Ph.D. Student, Department of Aerospace Engineering. Student Member AIAA.
†Professor, Department of Aerospace Engineering. Associate Fellow AIAA.
‡Associate Research Engineer, Applied Research Laboratory.

These examples demonstrate that in many instances the control of an aircraft with other than the primary controls is possible, even with extensive structural damage.[5–11] Unfortunately, similar incidents are likely to occur again in the future, and thus, it is important to understand better alternative control strategies that might improve the survivability of an aircraft in such situations. The accident of United Flight 232 and the attack on the DHL flight from Baghdad have demonstrated that the chances of survival increase significantly when flight crews are exposed to such scenarios through simulator and flight training and, possibly, even through safety briefings. Before their accident, at least one crew member of United Flight 232 had practiced unconventional control strategies in the aftermath of the loss of a Japan Airlines Boeing 747 that had lost its hydraulics.[6] Likewise, previous to their incident, the DHL crew had attended a safety seminar in which the captain of that fateful United flight had talked about his experience and possible control strategies for dealing with a total hydraulics failure.[2]

A considerable amount of research has been performed that explores control strategies of large transport aircraft with thrust-only control.[7–11] Of great help are the flight-control systems of modern transport aircraft that can be reconfigured to achieve a propulsion-controlled aircraft. In extensive simulator and flight test, alternative control strategies have been tested and assessed.

To explore alternative control strategies, a reasonably fast and accurate method for assessing the controllability of a transport aircraft in longitudinal and lateral–directional motion has been developed.[12–14] The primary emphasis of the method is on the dynamics of the motion of an aircraft with small initial disturbances and transients about its initial steady-state flight condition and on finding control strategies that allow the successful maneuvering of a damaged aircraft. Besides being less time consuming than the trial and error processes that are employed when exploring alternative control strategies with flight simulators and/or flight tests, the simulation tool enables the study of more complex scenarios that involve changes to the aircraft configuration, such as the loss of the vertical tail.

The objective of the study presented herein is to explore the use of this method in developing piloting strategies for a large transport aircraft that has lost its vertical tail. After discussing the theoretical model, the simulation results of several basic maneuvers are introduced with an undamaged aircraft, as well as one with partially and fully disabled conventional controls. These cases serve as a reliability check of the theoretical method before discussing the controllability of the aircraft after the loss of the vertical tail. Although such a scenario does not necessarily have to lead to disaster, it is difficult to explore in flight-test or simulator training. Nevertheless, unconventional control strategies, especially differential thrust, have the potential of substantially increasing the survival chances. These strategies are intended to be a starting point for crew training to increase the chances of survivability of such an, fortunately unlikely, event.

Simulation Method

The equations of motion of a rigid aircraft are linearized using a small-perturbation approach about the initial unaccelerated and wings level steady state. The assumption that longitudinal motions do not have an effect on lateral–directional motions and vice versa allows the decoupling of the longitudinal and lateral–directional motions. Based on this dynamic aircraft model, the particular control issue of interest is then formulated as a closed-loop tracking problem. When optimal control theory is used, the tracking problem is solved, and the aircraft states are found for the prescribed optimal control law.

Of course, not every case dealing with loss of conventional control is manageable by using unconventional strategies. For this reason, certain practical limits must be imposed on the flight scenarios that can be investigated. For example, the possibilities of saving an aircraft in which the damage is followed by large excursions or transients from the steady-state flight condition are limited. In such cases, it is likely that the available control power is insufficient to recover the aircraft before it exceeds operational limits. Likewise, the assumptions inherent in the linear model employed break down in situations where the perturbations become too large.

Many of the control-system failure incidents that have occurred do so with little disturbance from the initial trimmed steady-state flight condition. A jammed control surface or a broken hydraulic line does not usually result in sudden change in the flight condition. Because of the absence of large transients in these situations, the available control power is more likely to be sufficient to regain and maintain trimmed flight. Nevertheless, the aircraft eventually must be maneuvered into a position for landing. In addition, atmospheric conditions, such as wind and turbulence, must be taken into account. It is found that in many situations, limited maneuvers that have small accelerations are sufficient for following a desired trajectory. In such cases, using the linearized equations of motion greatly simplifies the model, but still yields results that are useful in evaluating control strategies.

At this point, it should be noted that other assumptions, such as that of a rigid airframe, might limit the direct application of the results in certain situations. In addition, some coupling between the longitudinal and lateral–directional motions due to the control inputs is expected in reality. This interaction can be accounted for by using a six-degree-of-freedom model. It is reasonable, however, to investigate the longitudinal and lateral–directional motions separately before simulating an aircraft with six degrees of freedom to allow insight into the mechanisms of unconventional control strategies.

Optimal Control Theory

The objective of optimal control theory, which was used for the method described herein, is to determine a control law that results in a process that satisfies the physical constraints and, at the same time, minimizes (or maximizes) some performance criterion. The method has an advantage over inverse formulations in that solutions for control strategies can be found that minimize a user-defined performance index J. The control system designer can specify a desired time-dependent trajectory and place constraints on control amplitudes and state outputs. The control solutions are generally well behaved and coordinated such that rapid control inputs that may cause unnecessary accelerations of the aircraft are avoided.[15] Optimal control theory, therefore, can be used to find possible control strategies for performing flight maneuvers while minimizing a performance cost based on control deflections and motion variables.[15–19]

Tracking Problem Formulation

To apply optimal control theory toward finding a control solution, the control system is modeled in the time domain. The decoupled equations of motion supply two sets of equations that describe the three longitudinal or three lateral–directional degrees of freedom of aircraft motion. Either set takes on the following state-space form:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

Equation (1) describes a linear, time-invariant system with constant plant and control matrices, $\boldsymbol{A}$ and $\boldsymbol{B}$, respectively. The plant matrix contains the aircraft derivatives and the control matrix contains the control derivatives. The plant states $\boldsymbol{x}$ are driven by the control inputs $\boldsymbol{u}$. Equation (1) is the mathematical model of the aircraft in the time domain as is needed to find an optimal control solution.

In this research, the performance index, J, is written as

$$J = \frac{1}{2}[\boldsymbol{x}(t_f) - \boldsymbol{r}(t_f)]^T \boldsymbol{H}[\boldsymbol{x}(t_f) - \boldsymbol{r}(t_f)] + \frac{1}{2}\int_{t_0}^{t_f} \{[\boldsymbol{x}(t) - \boldsymbol{r}(t)]^T \boldsymbol{Q}[\boldsymbol{x}(t) - \boldsymbol{r}(t)] + \boldsymbol{u}(t)^T \boldsymbol{R}\boldsymbol{u}(t)\}\,\mathrm{d}t \tag{2}$$

It has three contributions. The first one is the terminal control problem, which tries to minimize the deviation of the final state of the system, $\boldsymbol{x}(t_f)$, from its desired final value, $\boldsymbol{r}(t_f)$. Each terminal state error is weighted separately with the terminal weight matrix $\boldsymbol{H}$. The integral term consists of the sum of the performance measures from the tracking problem and the minimum control-effort problem. The tracking problem is to maintain the system state $\boldsymbol{x}(t)$ as close as

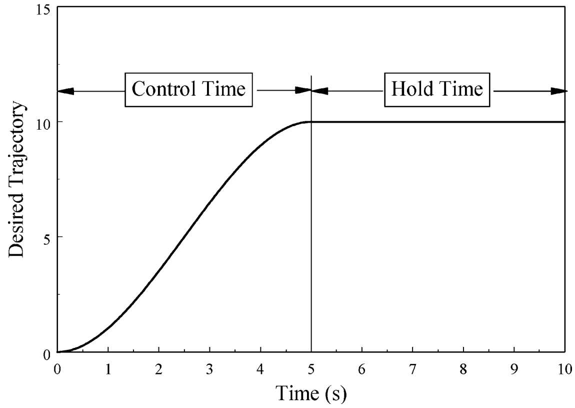

Fig. 1 Control time and hold time of desired state trajectory.

possible to a desired state $\boldsymbol{r}(t)$, in the time interval $[t_0, t_f]$. Figure 1 shows an example of a state that is to be tracked. The desired trajectory $\boldsymbol{r}(t)$ is defined by the user with a third-order polynomial between the desired initial and final states over a control interval. During the hold time that follows the control-time interval, the desired steady-state trajectory remains constant. The third-order polynomial for the desired trajectory results in much better behaved control solutions with less erratic control inputs than does a simple step command.[18] The minimum control-effort problem is to transfer a system from its initial state $\boldsymbol{x}(t_0)$ to its final state $\boldsymbol{x}(t_f)$, with a minimum expenditure of control input. The weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ allow the tracking of single states and the usage of control inputs, respectively, to be individually emphasized.

The optimal control law can be determined by solving the first-order differential equations for the Riccati matrix $\boldsymbol{K}$ and the command signal $\boldsymbol{s}(t)$ (see Ref. 19),

$$\dot{\boldsymbol{K}}(t) = -\boldsymbol{K}(t)\boldsymbol{A} - \boldsymbol{Q} - \boldsymbol{A}^T\boldsymbol{K}(t) + \boldsymbol{K}(t)\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{K}(t) \tag{3}$$

$$\dot{\boldsymbol{s}}(t) = -\{\boldsymbol{A}^T - \boldsymbol{K}(t)\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\}\boldsymbol{s}(t) + \boldsymbol{Q}\boldsymbol{r}(t) \tag{4}$$

Its matrix and nonlinear form make an analytical solution of the Riccati equation (3) difficult. Often, the time derivative of $\boldsymbol{K}(t)$ is set equal to zero, and it is solved simply as a steady-state problem. This method will usually give an answer that is sufficient, especially if the desired trajectory $\boldsymbol{r}(t)$ is constant. Optimal control law solutions of damaged transport aircraft as presented herein, however, have been found to be better behaved when Eqs. (3) and (4) are numerically integrated backward in time from t_f to t_0 (Ref. 13). The final values for $\boldsymbol{K}_f$ and $\boldsymbol{s}_f$ are given by

$$\boldsymbol{K}_f = \boldsymbol{H} \tag{5}$$

$$\boldsymbol{s}_f = -\boldsymbol{H}\boldsymbol{r}_f \tag{6}$$

The numerical integration of $\boldsymbol{K}(t)$ and $\boldsymbol{s}(t)$ is performed using a fourth-order Runge–Kutta scheme. The optimal feedback state equation $\boldsymbol{x}^*(t)$ is computed using

$$\dot{\boldsymbol{x}}^*(t) = [\boldsymbol{A} - \boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{K}]\boldsymbol{x}^*(t) - \boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{s}(t) \tag{7}$$

The optimal control law with which these states are reached equates to

$$\boldsymbol{u}^*(t) = -\boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{K}\boldsymbol{x}^*(t) - \boldsymbol{R}^{-1}\boldsymbol{B}^T\boldsymbol{s}(t) \tag{8}$$

As apparent from Eqs. (3–8), the solution of the optimal control law for a desired trajectory depends on the choice of weighting matrices $\boldsymbol{H}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$.

Analysis of Aircraft Control

The method thus far described was implemented on a personal computer with a program that allows the exploration of control strategies of damaged transport aircraft. Figure 2 shows the basic

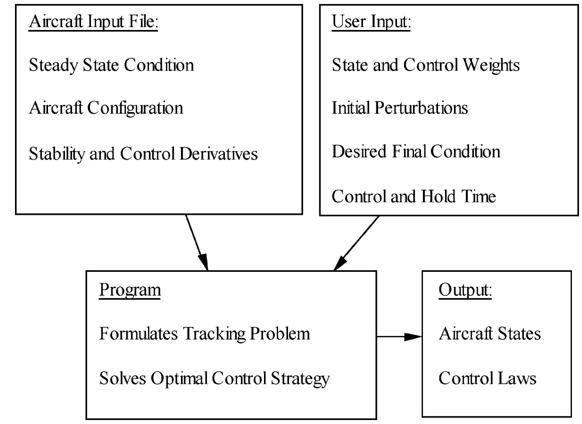

Fig. 2 Basic structure of file and screen input, processing, and output of simulation program.

structure of the program. Information about the aircraft configuration, such as its stability and control derivatives, is read from an input file. Also, additional information, such as the state and control weights, initial perturbations, desired final states, and control and hold times, is input through a graphical user interface. This interface allows quick and simple changes of the different input parameters. The program then finds a control law for the tracking problem as formulated in the preceding section and displays the computed states and control law on the screen. The user is then able to alter the outcome by changing the state and control weights or the simulation times. An increase in a specific state weight causes its deviation from the desired trajectory to be penalized more severely. Accordingly, a large control weight restrains the usage of that particular controller. The speed of the program, along with its uncomplicated input, allows different scenarios to be examined relatively quickly.

The solution is derived by applying the method of optimal control and is only optimal in the sense that it satisfies certain boundary conditions and is not necessarily physically realistic. The suitability of a solution is left to the discretion of the investigator, who can tune it by adjusting the weighting matrices $\boldsymbol{H}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$. Hence, a real-time application of this method is, unlike with other control theories, somewhat problematic.[9,10] The derived control law is rather meant to be a starting point for further studies. It helps to answer the question of whether or not a control strategy is available that might allow a damaged aircraft to get safely to the ground in a given situation. Furthermore, it can uncover potential problems with proposed unconventional control strategies. Thus, it is highly suitable as a training tool, as well as an aid to decreasing the number of trial and error iterations that might be needed otherwise in investigations using simulator or flight test.

Simulation Results

Aircraft Model

The aircraft model used in the simulations whose results are presented herein is based on a large transport aircraft with twin engines that are underneath the wing. The model and the simulation method have been verified with an eigenvalue analysis and comparison with other existing dynamic-responses results.[20,21] No significant differences were found.[12,13] Several maneuvers were simulated with an undamaged aircraft, as well as with one that has only differential thrust available for control.[12–14] It can be concluded that the simulation results with either configuration appear to be in relatively good agreement with what can be expected from a large transport aircraft.

The stability and control derivatives used for the examples presented in this study are taken from Ref. 20 and are loosely based on the airframe of a Boeing 747. The values of landing and cruise configuration are listed in Tables 1 and 2, respectively. The sign convention is also in accordance with Ref. 20 and is summarized in Fig. 3.

Several simplifications have been made to model a large transport aircraft. For example, the simulation achieves roll control with

Table 1 Mass properties and stability and control derivatives of the undamaged aircraft in landing configuration at sea level

Parameter	Value
Property	
W	564,000 lb
M	0.2
I_{XX}	13.7×10^6slug · ft^2
I_{YY}	30.5×10^6slug · ft^2
I_{XZ}	0.83×10^6 slug · ft^2
I_{ZZ}	43.1×10^6 slug · ft^2
Span	196 ft
S	5500 ft^2
Longitudinal	
C_{L_U}	−0.22
C_{L_α}	5.67
$C_{L_{\dot\alpha}}$	6.7
C_{L_q}	5.65
$C_{L_{\delta e}}$	0.36
C_{D_α}	1.13
C_{m_U}	0.071
C_{m_α}	−1.45
$C_{m_{\dot\alpha}}$	−3.3
C_{m_q}	−21.4
$C_{m_{\delta e}}$	−1.4
Lateral–Directional	
C_{l_β}	−0.281
C_{l_p}	−0.502
C_{l_r}	0.195
$C_{l_{\delta R}}$	0.0
$C_{l_{\delta A}}$	0.053
C_{n_β}	0.184
C_{n_p}	−0.222
C_{n_r}	−0.36
$C_{n_{\delta R}}$	−0.113
$C_{n_{\delta A}}$	+0.0083
C_{Y_β}	−1.08
C_{Y_p}	0.0
C_{Y_r}	0.0
$C_{Y_{\delta R}}$	0.179
$C_{Y_{\delta A}}$	0.0

Table 2 Mass properties and stability and control derivatives of undamaged aircraft in cruise configuration at 20,000 ft

Parameter	Value
Property	
W	636636 lb
M	0.4
I_{XX}	18.2×10^6 slug · ft^2
I_{YY}	33.1×10^6 slug · ft^2
I_{XZ}	0.97×10^6 · slug · ft^2
I_{ZZ}	49.7×10^6 · slug · ft^2
Span	196 ft
S	5500 ft^2
Longitudinal	
C_{L_U}	0.13
C_{L_α}	4.4
$C_{L_{\dot\alpha}}$	7.0
C_{L_q}	6.6
$C_{L_{\delta e}}$	0.32
$C_{D_{\dot\alpha}}$	0.2
C_{m_U}	0.013
C_{m_α}	−1.0
$C_{m_{\dot\alpha}}$	−4.0
C_{m_q}	−20.5
$C_{m_{\delta e}}$	−1.3
Lateral–Directional	
C_{l_β}	−0.16
C_{l_p}	−0.340
C_{l_r}	0.130
$C_{l_{\delta R}}$	0.008
$C_{l_{\delta A}}$	0.013
C_{n_β}	0.16
C_{n_p}	−0.026
C_{n_r}	−0.28
$C_{n_{\delta R}}$	−0.100
$C_{n_{\delta A}}$	+0.0018
C_{Y_β}	−0.9
C_{Y_p}	0.0
C_{Y_r}	0.0
$C_{Y_{\delta R}}$	0.120
$C_{Y_{\delta A}}$	0.0

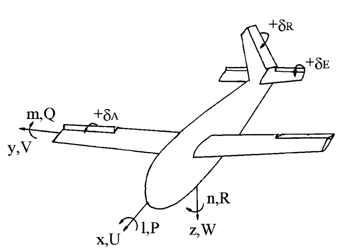

Fig. 3 Sign convention of velocities, moments, rates, and control-surface deflections.

Table 3 Vertical tail contributions to mass properties and stability derivatives

Parameter	Value	Parameter	Value
Property			
W_{VT}	4600 lb		
$I_{XX\mathrm{VT}}$	0.13×10^6 slug · ft^2	$I_{XZ\mathrm{VT}}$	0.32×10^6 slug · ft^2
$I_{YY\mathrm{VT}}$	1.47×10^6 slug · ft^2	$I_{ZZ\mathrm{VT}}$	1.47×10^6 slug · ft^2
Derivatives			
C_{l_β}	−0.068	C_{nr}	−0.242
C_{l_P}	−0.021	C_{y_β}	−0.451
C_{l_r}	0.071	C_{y_p}	−0.137
C_{n_β}	0.234	C_{y_r}	0.467
C_{n_p}	0.071		

ailerons, whereas modern large transports usually rely on a mix of inboard and outboard ailerons, as well as several spoilers, whose usage depends on flight speed and flap setting. Similar simplifications are made for pitch and yaw control that are modeled with single surfaces as well. Furthermore, the model does not include any lag in dynamic response of the actuators or engines. Although these simplifications can be significant in the absolute outcome of a particular aircraft, their absence is considered acceptable for exploring possible piloting strategies of a large transport aircraft without a vertical tail.

To explore alternative control strategies following the loss of the vertical tail, the overall inertias and derivatives of the undamaged aircraft were corrected with the contribution of the vertical tail. These contributions were estimated using the standard techniques[20] and are listed in Table 3.

The eigenvalues of the two configurations are listed in Table 4 for the aircraft with and without a vertical tail. Not very surprising is the loss of a stable Dutch-roll mode in the absence of the vertical tail. Especially in the case of the aircraft in cruise configuration, the vertical tail loss results in an unstable Dutch-roll mode that is only weakly oscillatory. Furthermore, the roll mode of this configuration becomes less stable.

Maneuvers with Conventional Control Strategies

To demonstrate the validity of the simulation method and its control laws, several maneuvers were simulated using the undamaged aircraft with conventional controls. Some of these results are shown in Figs. 4 and 5. The aircraft is in landing configuration, and the results are obtained using the derivatives listed in Table 1.

In the first example, for which the time histories of the states and control deflections are shown in Fig. 4, the aircraft banks from level flight to a desired bank angle of 15 deg. The control time chosen is 4 s. All state weights, with the exception of sideslip and bank angle, are chosen to be zero. The penalty for deviation from the desired trajectory is 50 times greater for bank than it is for sideslip angle.

Table 4 Eigenvalues of the modeled aircraft in its different configurations

	Landing configuration (sea level)		Cruise configuration (20,000ft)	
Modes	With vertical tail	Without vertical tail	With vertical tail	Without vertical tail
Phugoid	$-0.0058 \pm 0.1720i$	$-0.0065 \pm 0.1719i$	$-0.0010 \pm 0.1039i$	$0.0009 \pm 0.1039i$
Short period	$-0.4808 \pm 0.6087i$	$-0.4937 \pm 0.6235i$	$-3.708 \pm 0.6845i$	$-3.815 \pm 0.6990i$
Dutch roll	$-0.0714 \pm 0.7473i$	$+0.0385 \pm 0.4832i$	$-0.1246 \pm 1.0431i$	$+0.1446 \pm 0.0400i$
Spiral	-0.0434	-0.0548	-0.0171	-0.1491
Roll	-1.1460	-1.0908	-0.9392	-0.6790

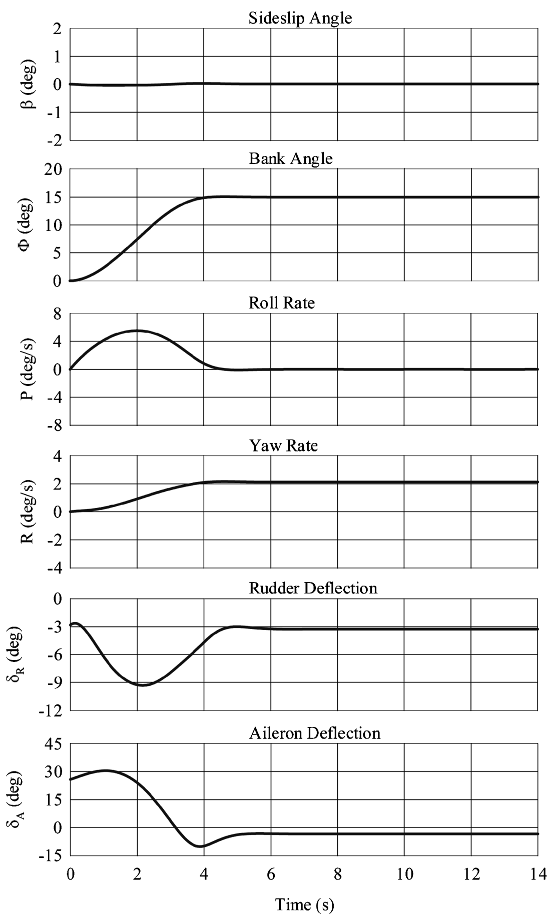

Fig. 4 States and control law for establishing 15-deg bank with aileron and rudder controls; landing configuration with a vertical tail.

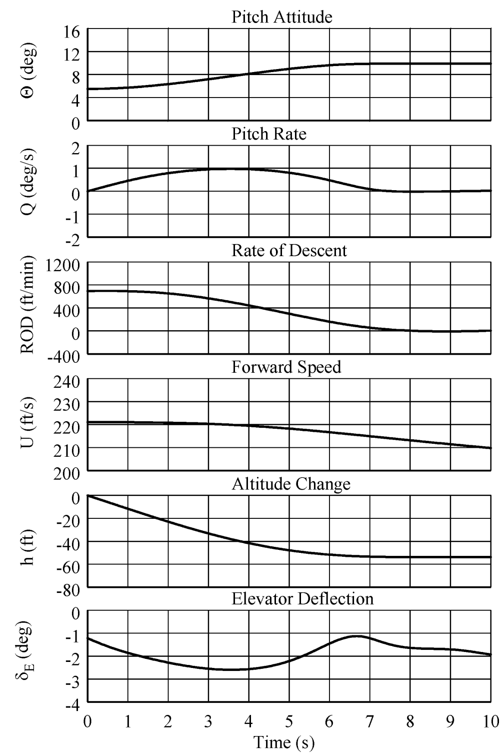

Fig. 5 States and control law for landing flare maneuver with elevator control; landing configuration with a vertical tail.

The entire maneuver is essentially flown in a coordinated manner with insignificant sideslip. Bank angle reaches the desired 15 deg within the control time interval. At the same time, the yaw rate reaches a new steady value of about 2.16 deg/s, which corresponds to the steady-state yaw rate with 15-deg bank and an airspeed of approximately 130 kt. Examining the computed optimal control law shows that the aileron is fully applied during the first 2 s to produce the necessary roll rate. After that, it is gradually reduced and even deflected in the opposite direction to stop the rolling motion. Once the rolling is stopped, the aileron is slightly reduced, but remains at a constant -4 deg after the steady bank angle is reached. This prevents the aircraft from continuing to roll into the turn due to its overbanking dynamics. Adverse yaw is compensated with a negative rudder deflection that peaks at -9 deg, when the aileron deflections are large and reduces to about -3 deg after steady banked flight is established. It should be noted that, according to the sign convention used, positive rudder results in a negative yawing moment.

The next validation example considers the landing flare maneuver, the phase during the landing approach at which the aircraft transitions from its descending flight path to a horizontal path just before touch down. For this analysis, it is assumed that the aircraft is positioned on final and has established a steady approach speed and glide slope of 3 deg. The aircraft descent rate is eventually reduced to zero by increasing pitch attitude. Figure 5 shows the predicted time histories of such a maneuver with elevator control and at constant engine thrust. With the exception of pitch attitude, all state weights are set to zero. The control time is set at 8 s. During the maneuver, the elevator induces the pitch rate needed to achieve the desired pitch angle. After the pitch angle is obtained, the elevator deflection is reduced to a value that allows maintaining the new aircraft pitch attitude. With the increasing of pitch angle, the aircraft rate of descent is reduced and the aircraft flight path levels out after about 8 s, as can be seen by the altitude. In addition, the flight speed decreases due to increased drag at the resulting higher angle of attack. Because the aircraft model relies on constant derivatives, the simulation does not capture the increased pitchdown moment of the aircraft as it enters ground effect. Nevertheless, the simulation essentially does capture the physics of the landing maneuver.

States and control inputs of the predicted maneuver using conventional controls, aileron, rudder, and elevator, appear to be quite realistic. For example, often airplanes have slightly crossed controls during a steady and coordinated turn, as is predicted at the end of the 15-deg banked turn maneuver. Results such as those presented are obtained with relative ease and do not require a great deal of intuition or analysis beforehand. The most important requirement is the correct specification of the desired final conditions. Defining state weights has also proven to be quite straightforward. In fact, most weights are set to zero for the majority of the maneuvers. The

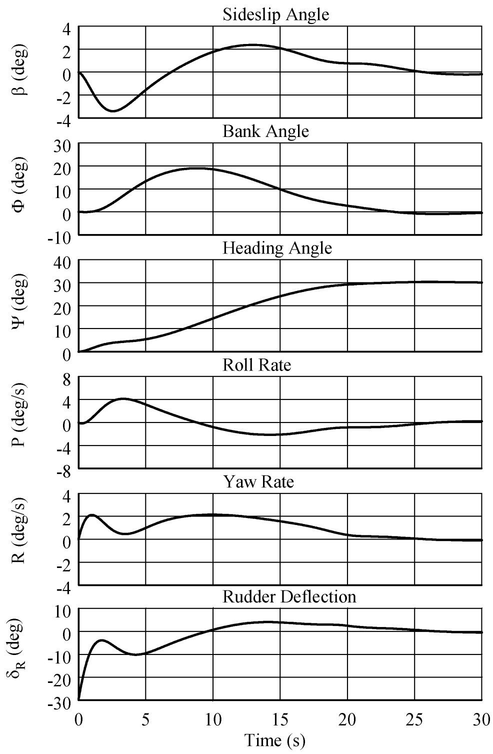

Fig. 6 States and control law for 30-deg heading change with rudder control; landing configuration with a vertical tail.

remaining weights are adjusted accordingly, during successive iterations, to avoid excessive or physically impossible control inputs.

Maneuvers with Unconventional Control Strategies

To further demonstrate the plausibility of the simulation approach and its results, the following two simulations with unconventional control strategies and undamaged airframe are considered. To save an aircraft successfully after damage to its control system has occurred, its flight path must first be stabilized and, then, it must be guided to a landing point and landed. As with the cases considered having no control damage, those presented here deal mainly with maneuvering the damaged aircraft to a desired heading and performing the landing flare.

To land, the aircraft must establish certain headings and, ultimately, needs to be aligned with the runway. Obviously, this maneuver can be simulated in several ways. One possibility is to split the maneuver into two parts: first, establishing a constant yaw rate for the aircraft and, then, restoring it to a wings-level, coordinated flight condition. Another possibility is to specify a desired heading and use control strategies that are usually not employed, such as excessive slipping. Of course, once on the new heading, the flight condition must be restored to a coordinated, wings-level flight condition. The latter method is used to obtain the time histories that are shown in Fig. 6 for an aircraft in the landing configuration; as represented by the values in Table 1, with only the rudder available as a controller. Although the output has been truncated after 30 s for better clarity, the control interval is 20 s, followed by 40 s of hold time. The state and control weights are all chosen to be zero with the exception of sideslip angle, bank angle, and heading angle. The heading follows the prescribed trajectory and, after about 20 s, is close to the desired final value of 30 deg. The maneuver is started with full negative rudder deflection, which causes a positive yaw rate and a negative slip angle. Consequently, the bank angle increases due to dihedral effect and yaw–roll coupling. As soon as roll rate increases, the rudder deflection is reduced such that the sideslip is decreased. A temporary increase of rudder becomes necessary, however, to compensate for a sudden reduction in yaw rate. After the yaw rate recovers, mainly because of the increasing bank angle, the rudder deflection is further reduced and eventually becomes positive, as does the sideslip. In addition, the roll rate decreases and becomes slightly negative after the maximum bank angle of about 19 deg is reached. The rudder deflection and sideslip angle remain slightly positive until level flight with the new heading is established after about 25 s. Thus, the time required for the maneuver is no longer than that needed with the conventional controls.[13]

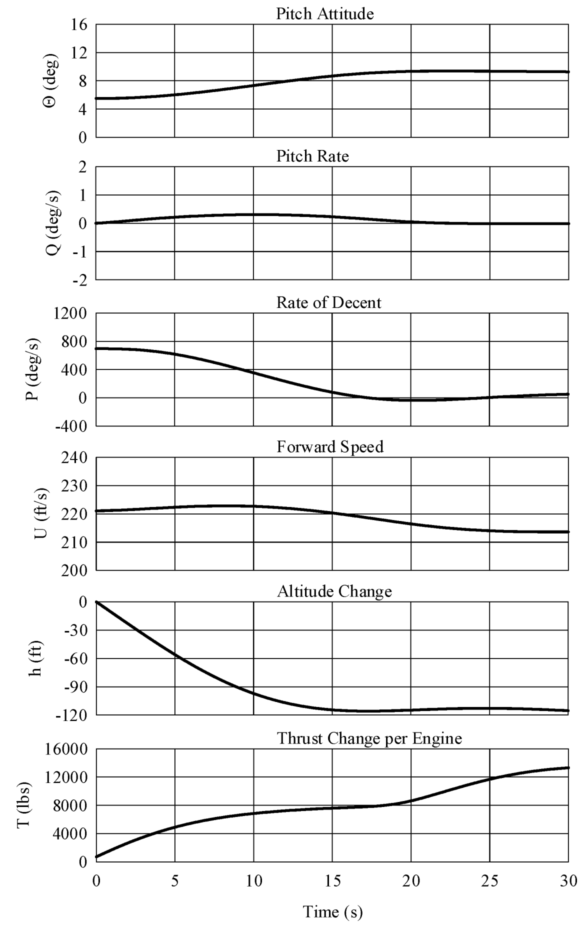

Fig. 7 States and control law for landing flare maneuver with thrust-only control; landing configuration with a vertical tail.

Another interesting and realistic scenario is the loss of all conventional controls, for example, due a hydraulic failure. In this case, thrust-only control is the only option available to control and attempt to land the aircraft. Previous studies have demonstrated the feasibility of thrust-only control.[7–14] The time history of the states and the optimal control law for the flare maneuver using only thrust control is shown in Fig. 7. As in the case using conventional controls, the pitch attitude is desired to be increased by 4.4 deg. Accordingly, all state weights except pitch attitude are set to zero. With the engines underneath the wing, a thrust increase results in a pitchup moment with an eventual lower speed. A thrust reduction results in a pitchdown moment and a speed increase due to the steeper flight path. To account for the lowered maneuverability with thrust control, the control time was doubled from the undamaged case to 16 s. By the increase of the thrust of each engine by about 8000 lb (1800 N) from their initial approach settings, a noseup pitching moment is generated and the pitch attitude increased. To reduce pitch rate, the thrust is kept constant at that level, only to be increased further once the desired final pitch angle is reached after about 16 s. At that point, the pitch rate approaches zero. The sink rate and altitude show that the horizontal flight path desired for touchdown is reached after about 17 s, which is about twice the time required in the undamaged control situation. In addition, the altitude needed to flare is about twice that required in the case with full elevator control. The drop in flight speed during the maneuver is not as pronounced in this case as it was in the preceding one, but is acceptable for an emergency landing.

Unconventional Control Strategies after Loss of the Vertical Tail

The aircraft without vertical tail simulations presented here are separated into two different situations. The first ones deal with the recovery from an initial upset during which the primary objective is the reestablishment of steady, level flight. The second set, which is similar to the cases that were discussed in the earlier section, deals with establishing a desired heading to maneuver the aircraft to landing. For these simulations, the stability derivatives of the two configurations in Tables 1 and 2 are adjusted using the values listed in Table 3.

After loss of the vertical tail, longitudinal stability and control becomes mainly an issue of having enough pitch control to compensate for the more forward location of the center of gravity of the aircraft without the vertical tail. Pitch control can be enhanced with the thrust of engines that are mounted underneath the wing below the vertical center of gravity location. For these reasons, no longitudinal cases are presented with these simulations.

As shown in Fig. 8, the dynamic response of the aircraft without a vertical tail in landing configuration due to a lateral gust of 30 ft/s from the right side, or roughly a 4-deg sideslip, is divergent. Roll rate and yaw rate increase rapidly to the left, and the right sideslip angle doubles within approximately 5 s. After 10 s, the sideslip angle exceeds 20 deg and bank angle 80 deg. The airplane is unstable, and its dynamic response clearly exceeds the small-perturbation approximation of the method but is also in line with the unstable Dutch-roll mode indicated in Table 4.

An example of the states and control laws that are required to recover from the same initial disturbance using only aileron control for the aircraft in its cruise configuration is shown in Fig. 9. Although the states remain in acceptable limits and steady, level flight is essentially regained after about 50 s, the required control law is unrealistic, and a successful recovery is doubtful because it requires an initial left aileron deflection of 60 deg to induce the roll rate and subsequent bank angle that eventually stops the right sideslip motion. Such an extreme aileron deflection exceeds the actual possible range and is beyond the assumed linearity of aileron effectiveness. The result demonstrates the difficulties with recovering using only aileron control. Nevertheless, no better control laws could be found, despite increasing the aileron weight considerably, relaxing the allowable states, or by significantly increasing the simulation time.

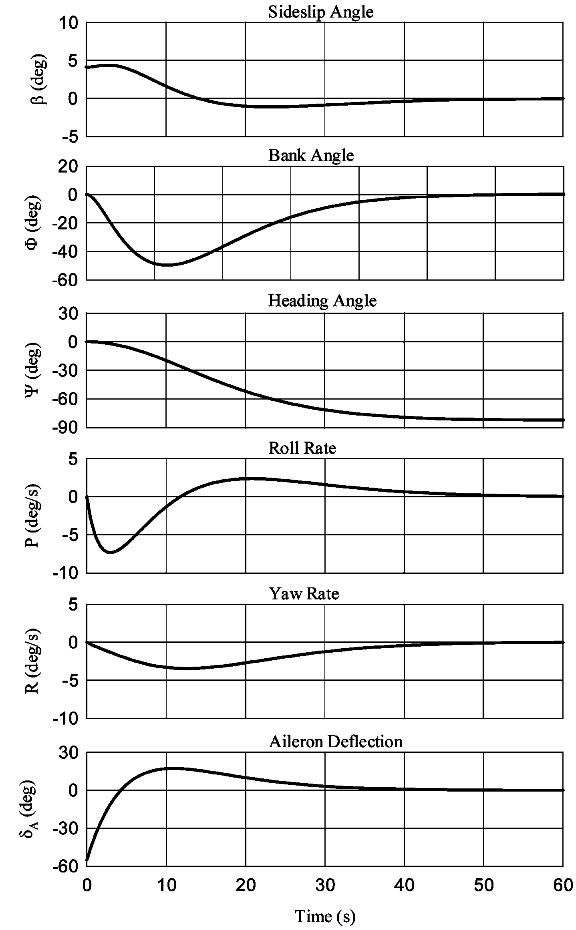

Fig. 9 States and control law during recovery from a sideslip upset of 4 deg with aileron; cruise configuration without vertical tail.

The same maneuver is more successful when differential thrust is used as an additional controller for the same configuration as in the preceding case. The simulation results of this are shown in Fig. 10. Initially, the thrust of the port engine is increased by about 20,000 lb (4500 N), almost doubling the cruise setting, while the starboard engine is throttled back to half the cruise setting. The resulting right yawing moment limits the maximum yaw rate to the left to less than 1.9 deg/s. Simultaneously, full left aileron induces a roll rate and the subsequent left bank angle to stop the sideslipping of the aircraft to the right. Soon, however, the aileron is reversed to stop the rolling motion and level the aircraft. Thrust of the port engine is also reduced to levels slightly below the cruise setting to compensate for a small negative sideslip. Steady and level flight is achieved after approximately 40 s, after a heading change of almost 45 deg to the left of the original flight path.

After the aircraft has been recovered from the initial disturbance, it becomes important to maneuver it to the desired point of landing. Whereas the previous maneuver was mostly concerned with reestablishing steady, level flight, the following ones deal with establishing the proper flight directions for landing and requires a series of heading changes, followed by the final approach and touchdown.

A heading-change maneuver using aileron and differential thrust control is shown in Fig. 11 for the aircraft in cruise configuration without a vertical tail. The desired final heading is 30 deg to the right of the original flight path. The deflection of the right aileron and the thrust increase of the two engines initiate the banking and yawing to yield a somewhat steady turn after about 8 s. Just as with the earlier maneuver, the control weight of the starboard engine has been adjusted so that only small inputs are required for the maneuver. During the following 10 s, overbanking is avoided with about 5-deg left aileron, and the bank angle remains relatively steady at about

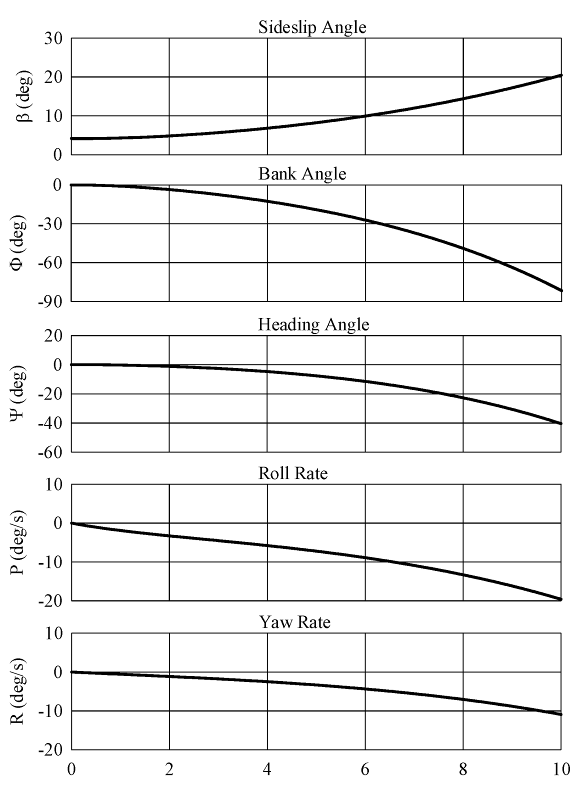

Fig. 8 Dynamic response after a sideslip upset of 4 deg; cruise configuration without vertical tail.

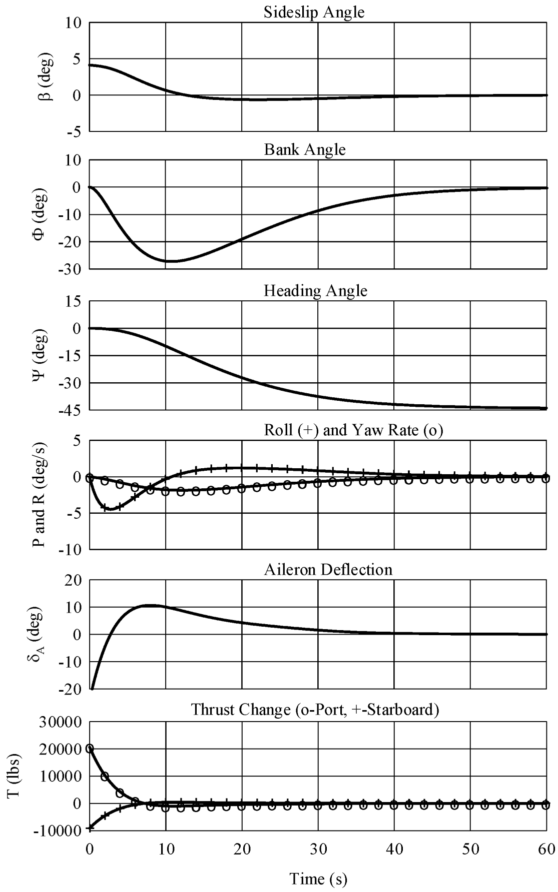

Fig. 10 States and control law during recovery from a sideslip upset of 4 deg with aileron and thrust control; cruise configuration without vertical tail.

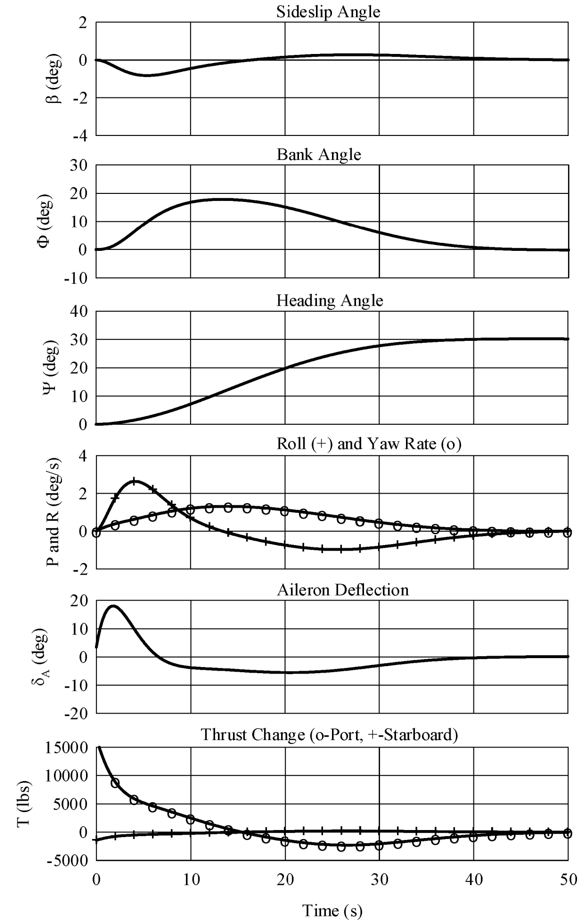

Fig. 11 States and control law during 30-deg heading change with aileron and thrust control; cruise configuration without vertical tail.

17 deg. A reduction in left aileron decreases the bank angle after about a 25-deg heading change. The new heading and steady, level flight are achieved after 40 s.

The simulation results of the same maneuver are shown in Fig. 12, but in this case with the control weight of the aileron more relaxed than in the preceding case. Throughout the maneuver, the aircraft states behave similarly, although a small roll rate occurs within the first few seconds to the left. Furthermore, sideslip to the left peaks at approximately a half of a degree larger value during the first 10 s. The most apparent differences are the reduced thrust levels and the aileron-deflection schedule, which is initially deflected fully to the left, but changes to full right within the first 4 s. After this, the ailerons follow the control law of the case already discussed. The initial full left deflection of the aileron is highly counterintuitive, because one would normally expect that aileron right would be needed to establish a turn to the right. Without the vertical tail, however, the aircraft has a much more negative roll-induced yawing moment-stability derivative C_{n_p}, or adverse yaw, -0.097 compared to -0.026 with the vertical tail. The left aileron deflection takes advantage of this strong adverse roll–yaw coupling of the aircraft and helps to achieve the desired yaw with an initial roll rate to the left. Note that the yaw derivative due to the deflection of ailerons, $C_{n_{\delta A}}$, is in fact slightly positive, as shown in Table 2.

As a consequence of losing the vertical tail, it is also likely that the hydraulic control lines would be severed, and eventually, all conventional controls might be lost. In that case, the hydraulic-independent thrust control system is the only remaining means of control. Because the bleeding of the hydraulics may take some time, full aileron control can be assumed for the recovery from the initial upset, but eventually thrust becomes the predominant controller for the subsequent maneuvers to set up the aircraft for landing.

The results of a heading change of 30 deg with the aircraft in cruise configuration and without a vertical tail using thrust-only control are shown in Fig. 13. Full differential thrust is used and, of course, limited between idle and full thrust. To initiate the heading change after the damage, thrust of the port engine is increased by about 7500 lb (1700 N) and that of the starboard ones is decreased by the same amount. The resulting sideslip angle to the left and the strong dihedral effect of the aircraft produce the required right roll rate, and soon, the differential thrust levels are reversed to arrest the roll rate. After about 15 s, another reversal of differential thrust is used to maintain an almost steady turn rate. Those thrust levels are eventually returned to their steady-state values toward the end of the maneuver as the new heading is approached. Fully steady, level flight is achieved after 60 s.

The simulation results of the same maneuver using thrust-only control for the aircraft in its landing configuration are shown in Fig. 14. It is also the same maneuver and configuration as already introduced in Fig. 6 for the rudder-only control of the aircraft with vertical tail. The initial thrust values are very similar to the earlier cruise-configuration, thrust-only-control case, as shown in Fig. 13. A slightly larger sideslip angle induces the desired roll rate. Thrust is kept relatively constant until a bank angle of roughly 10 deg is reached after 10 s. After this point, the thrust is reduced to a lower value that is maintained for 22 s into the turn, after which the thrust levels are reversed to reduce the bank angle and level out the aircraft to its new heading. Again, the desired roll rate is achieved with sideslip and the strong yaw–roll coupling of this configuration. The maneuver requires approximately the same time as in the earlier discussed case and twice the time for the intact airframe maneuver discussed in Fig. 6. The most apparent difference is the oscillatory behavior of the states and thrust input of the landing configuration. The period of approximately 13 s agrees with the slightly unstable Dutch roll of this configuration listed in Table 4. Although this shows that a control strategy does indeed exist, its implementation

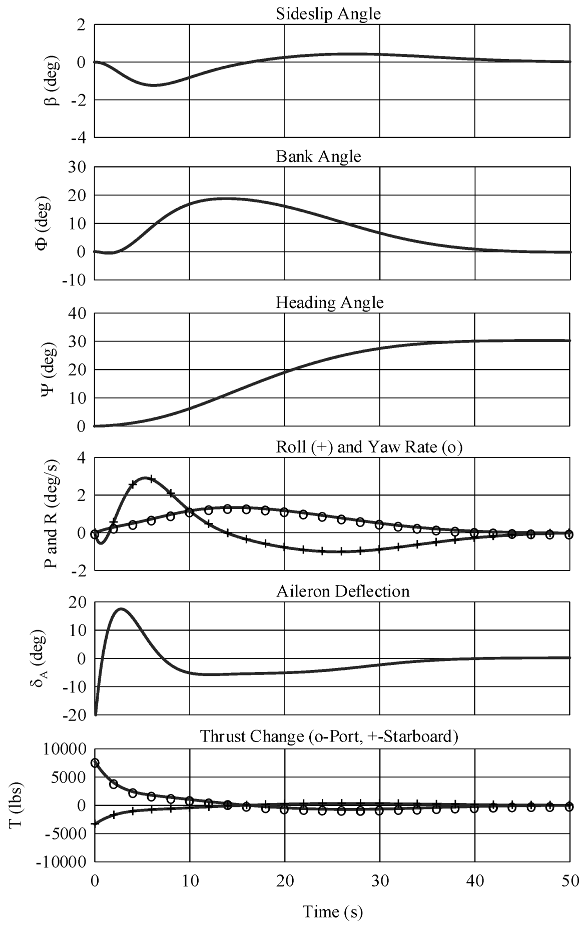

Fig. 12 Alternative states and control law during 30-deg heading change with aileron and thrust control; cruise configuration without vertical tail.

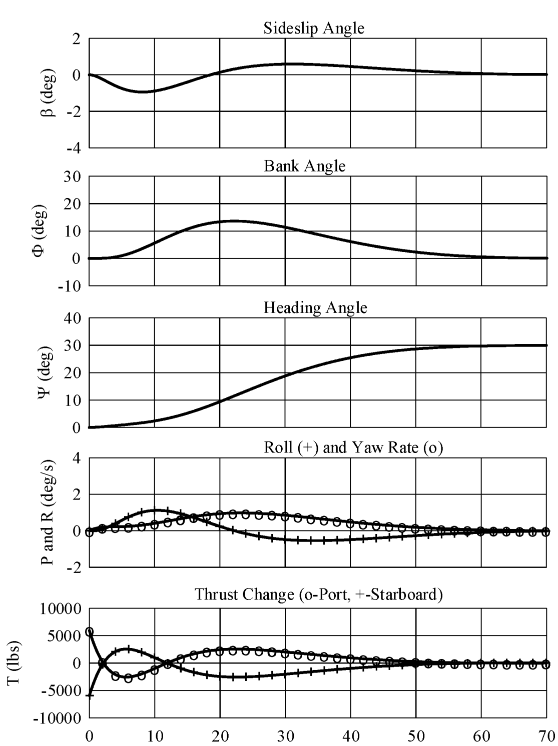

Fig. 13 States and control law during 30-deg heading change with thrust-only control; cruise configuration without vertical tail.

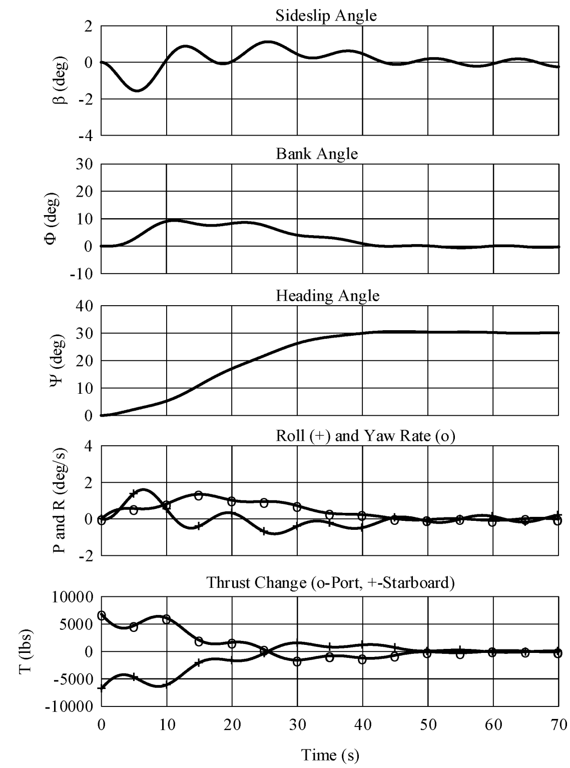

Fig. 14 States and control law during 30-deg heading change with thrust-only control; landing configuration without vertical tail.

by a crew might be problematic. Nevertheless, it forms the basis for more practical control strategies.

Discussion

The sample cases presented, along with other cases that have been examined, demonstrate that an aircraft with damaged primary controls can have limited, but usable, controllability. Unconventional strategies, such as differential thrust, work quite well in many cases, such as when establishing new headings, establishing steady turns, restoring level flight, and flaring for landing. This capability, of course, depends on the particular type of aircraft and upset in question.

Depending on the damage scenario, it is beneficial to choose the configuration of the aircraft having the most favorable control properties. For example, in its landing configuration the aircraft possesses a relatively large coupling between the lateral and directional motions. This is important when trying to control lateral–directional motions with only rudder or differential thrust, although care has to be given to controlling the Dutch-roll mode after the loss of the vertical tail. Another advantage of the landing configuration is that the lower speed keeps the system momentum low, although the relatively high thrust required for level flight might be a disadvantage when using thrust-only control.

It is also clear that the control power is not always sufficient to reach a desired state. A good example of this is the upset recovery of the aircraft without a vertical tail in that it appears impossible to recover the aircraft from the sideslip disturbance using ailerons only. After adding differential thrust, a successful recovery becomes possible.

When controlling the aircraft without vertical tail, differential thrust must provide the stability and control that are otherwise accomplished with the vertical tail and rudder. For example, in the beginning in the initial upset case that is shown in Fig. 10, differential thrust produces the restoring weathercocking moment that would be, otherwise, generated by the vertical tail. Furthermore, the yawing-moment inputs of differential thrust are very similar to rudder inputs that would be made in the case of the intact aircraft. The most apparent similarities are for the cases of 30-deg heading change with only rudder control and an intact airframe, shown in Fig. 6, and those with only thrust control after loss of the vertical tail, in Figs. 13 and 14. In all three cases, an initially large right yawing moment induces a yaw rate and a subsequent left sideslip angle that result in a roll rate to the right, due to the pronounced dihedral effect. After this, the yawing moment is drastically reduced to limit the sideslip. In general, the state histories are similar, although the amplitudes for the damaged airframe states are roughly half those values of the intact one. Consequently, the maneuver with the damaged airframe requires more time. In addition, the aircraft in landing configuration and without a vertical tail requires more attention to control the oscillatory-unstable Dutch roll.

Somewhat surprising and counterintuitive is the control strategy suggested by the results presented in Fig. 12, where left aileron is used to induce a right yawing moment for the following nearly coordinated right turn. The airframe without a vertical tail, however, has a relatively strong adverse roll–yaw coupling, which means that a roll to the left produces a distinct right yawing moment and vice versa. This behavior certainly merits further investigation because it has great potential to enhance alternative control strategies.

Intuitively, any control input should be applied carefully to not introduce large perturbations and transients. In addition, note that some of the system responses are characterized by large time lags, which might limit controllability considerably. A slow system response might be hard to detect, for example, or the time for jet engines to spool up might be longer than what is required. Further complications may arise because it is difficult or impossible for the crew of a damaged aircraft to observe the states needed to identify the correct actions. For example, yaw rate is very hard to distinguish from a heading change rate. The results shown demonstrate how much they can differ. In addition, malfunctioning instrumentation, which may be due to the damage, can aggravate the problems in evaluating the flight attitude. Clearly, the crew needs unambiguous indications of the aircraft states to take the appropriate action for properly controlling a disabled aircraft.

Conclusions

The dynamics of a transport aircraft are modeled using linearized equations of motion and a set of generalized control terms for elevators, ailerons, rudders, and engine thrust. An optimal control problem is formulated using the initial and desired final flight conditions of the longitudinal or lateral–directional motions. When the transient Riccati equation is solved, the optimal control law for a given task is found that minimizes a performance index consisting of control effort, tracking error, and terminal state error.

The method has been validated using an eigenvalue analysis and several open- and closed-loop simulations to predict the dynamics of a large transport aircraft with twin engines. The comparison of the predicted dynamics with those from other sources shows good agreement, which supports the validity of the simulation results. Additionally, several scenarios with a partially disabled control system have been investigated. The optimal control solutions appear to agree well with the experience in such cases.

Simulations with an aircraft model that has been adjusted for the loss of its vertical tail reveal that controllability can be achieved or augmented with differential thrust. The addition of differential thrust makes the recovery from an initial upset possible, for which aileron control is insufficient. Furthermore, heading changes for directing the aircraft to a landing point are shown to be possible using ailerons and differential thrust, as well as only differential thrust. These maneuvers demonstrate that the control of a transport aircraft is possible even after the loss of its vertical tail when employing alternative control strategies, such as differential thrust.

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